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SUPERGRAVITY AND THE $b \rightarrow s, \gamma$ DECAY

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We evaluate the branching ratio $\text{BR}(b \rightarrow s, \gamma)$ in the minimal supersymmetric standard model (MSSM), determining the corresponding phenomenological restrictions on two attractive supergravity scenarios, namely minimal supergravity and a class of models with a natural solution to the μ problem. We have included in the calculation some one-loop refinements that have a substantial impact on the results. It is stressed the fact that an eventual improvement of the experimental bounds of order 10^{-4} would strengthen the restrictions on the MSSM dramatically. This would be enough to discard these supergravity scenarios with $\mu < 0$ if no discrepancy is found with the standard model prediction, while for $\mu > 0$ there will remain low-energy windows.

1 The $b \rightarrow s, \gamma$ decay in SUGRA models

The new experimental data available¹ place stringent upper and lower bounds on the branching ratio (BR) for the process $b \rightarrow s, \gamma$: $1 \times 10^{-4} < \text{BR}(b \rightarrow s, \gamma) < 4 \times 10^{-4}$. On the other hand it is well known that this process has the potential to put relevant constraints to physics beyond the Standard Model (SM). In particular, we want to study the prediction that for this BR give two different supergravity (SUGRA) models whose particle content at low energies is that of the Minimal Supersymmetric Standard Model (MSSM)^a. That is, we will evaluate the expression (in units of the BR for the semileptonic b decay):

$$\frac{\text{BR}(b \rightarrow s\gamma)}{\text{BR}(b \rightarrow ce\bar{\nu})} = \frac{|K_{ts}^* K_{tb}|^2}{|K_{cb}|^2} \frac{6\alpha_{QED}}{\pi} \times \frac{[\eta^{16/23} A_\gamma + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) A_g + C]^2}{I(z)} F \quad (1)$$

^aA detailed description of the calculation is given elsewhere².

where $z = \frac{m_c}{m_b}$, $\eta = \frac{\alpha_s(M_W)}{\alpha_s(m_b)}$, $I(z)$ is the phase space factor, C stands for the leading logarithmic QCD corrections, and F contains NLO effects. Finally, $A_{\gamma,g} = A_{\gamma,g}^{SM} + A_{\gamma,g}^{H^-} + A_{\gamma,g}^{X^-}$ are the coefficients of the effective operators for the $bs\gamma$ and bsg interactions³; in our case we consider as relevant the contributions coming from the SM diagram (top quark and W^-) plus those with top quark and charged Higgs, and stops/scharmions and charginos running in the loop. It is interesting to note that both $A_{\gamma,g}^{SM}$ and $A_{\gamma,g}^{H^-}$ have always the same sign, giving therefore a total amplitude which is bigger than the SM one. However the presence of the chargino contribution which, for a wide range of the parameter space has opposite sign to that of the other two amplitudes, softens this effect. This is not enough, in general, to lower the total BR below the SM prediction, although in some cases it might be possible to have such big values for $A_{\gamma,g}^{X^-}$ as to drive the total BR even below the CLEO lower bound.

The MSSM is defined by the superpotential, W , and the form of the soft supersymmetry breaking terms. W is given by

$$W = \sum_i \{h_{u_i} Q_i H_2 u_i^c + h_{d_i} Q_i H_1 d_i^c + h_{e_i} L_i H_1 e_i^c\} + \mu H_1 H_2 \quad , \quad (2)$$

where i is a generation index, Q_i (L_i) are the scalar partners of the quark (lepton) SU(2) doublets, u_i^c, d_i^c (e_i^c) are the quark (lepton) singlets and $H_{1,2}$ are the two SUSY Higgs doublets; the h -factors are the Yukawa couplings and μ is the usual Higgs mixing parameter. The soft breaking terms have the form

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & \frac{1}{2} M \lambda_a \lambda_a - \sum_j m^2 |\phi_j|^2 - \sum_i A [h_{u_i} Q_i H_2 u_i^c + h_{d_i} Q_i H_1 d_i^c \\ & + h_{e_i} L_i H_1 e_i^c + \text{h.c.}] - [B \mu H_1 H_2 + \text{h.c.}] \quad , \end{aligned} \quad (3)$$

where a is gauge group index and λ_a are the gauginos. m , M , A , B are the scalar and gaugino masses and the coefficients of the trilinear and bilinear scalar terms, respectively. They are assumed to be universal at the unification scale M_X .

The aim of our calculation is to obtain the restrictions on the MSSM from the $b \rightarrow s, \gamma$ decay in the two SUGRA scenarios defined by:

SUGRA I

This is just the minimal SUGRA theory. It is defined by a Kähler potential $K = \sum_j |\phi_j|^2$ and a gauge kinetic function $f_{ab} = \delta_{ab}$, so that all the kinetic terms are canonical, whereas the superpotential W is assumed to be as in eq. (2), μ being an initial parameter. Then all the soft terms are automatically universal and the coefficients A and B are related to each other by the well known relation $B = A - m$. This scenario has a serious drawback, namely

the unnaturally small (electroweak) size of the initial μ parameter in the superpotential (often known as μ problem); that leads us to the next scenario.

SUGRA II

Recently, there have appeared several attractive mechanisms to solve the μ problem⁴ that, quite remarkably, lead, in the presence of a Kähler potential as in minimal SUGRA, to a similar prediction for the value of B , namely $B = 2m$. This can be achieved also if, as suggested by many superstring constructions, the kinetic terms are non-universal⁵. From now on we will refer (any of) these scenarios as SUGRA II. The corresponding MSSM emerging from them is as in minimal SUGRA except for the value of B .

So our starting point is the MSSM with initial parameters $\alpha_X, M_X, h_t, h_b, h_\tau, \mu, m, M, A, B$. By consideration of one of the two previous SUGRA theories we eliminate the B parameter, and then we demand consistency with all the experimental data, that is:

i) Correct unification of the gauge coupling constants

This can only be achieved when the renormalization group equations (RGE) of the gauge couplings are taken at two-loop order. For consistency, all the supersymmetric thresholds (and the top quark one) have to be taken into account in the running in a separate way. Therefore, given a choice of the initial parameters, an iterative process is necessary to achieve an agreement between the resulting prediction for $\alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z)$ and their experimental values⁶.

ii) Correct masses for all the observed particles.

Concerning the masses of the fermions, the boundary conditions for Yukawa couplings of the top and bottom quarks and the tau lepton, h_t, h_b, h_τ , have to be chosen so that the experimental masses are properly fitted (note that the running masses $m_{t,b,\tau}(Q)$ do not coincide with the physical (pole) masses $M_{t,b,\tau}$). Notice also that, as mentioned above, the masses in the $\text{BR}(b \rightarrow s, \gamma)$ are the running masses at the electroweak scale.

iii) Masses for the unobserved particles compatible with the experimental bounds⁷.

iv) Correct electroweak breaking, i.e. $M_Z = M_Z^{exp}$.

The vacuum expectation values (VEVs) of the two Higgses, $v_1 = \langle H_1 \rangle$, $v_2 = \langle H_2 \rangle$ (upon which M_Z depends) are to be obtained from the minimization of the Higgs potential. The tree level part of this in the MSSM has the form $V_o = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + 2\mu B H_1 H_2 + \frac{1}{8}(g^2 + g'^2)(|H_2|^2 - |H_1|^2)^2$, where all the parameters are understood to be running parameters evaluated at the renormalization scale Q . By a suitable redefinition of the phases of the fields it is always possible to impose $v_1, v_2 > 0$. As was clarified some time ago⁸, V_o and the corresponding tree level VEVs $v_1^o(Q), v_2^o(Q)$ are strongly Q -dependent. In

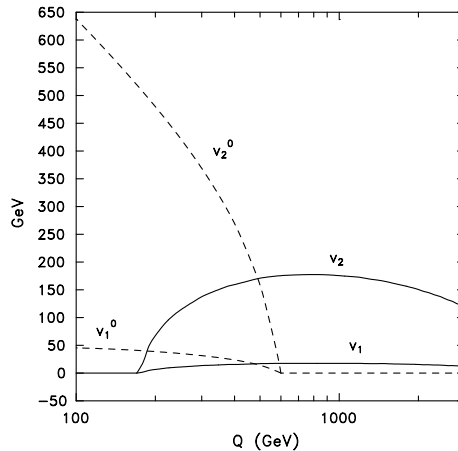


Figure 1: Plot of $v_1 \equiv \langle H_1 \rangle$, $v_2 \equiv \langle H_2 \rangle$ vs the Q scale between 100 GeV and 3 TeV for the model defined by $m = M = 300$ GeV, $A = 317$ GeV, $B = A - m$, $\mu = 403.76$ GeV, $h_t = 0.568$, $h_b = 0.063$, $h_\tau = 0.072$, $\alpha_X = 0.0404$ (all quantities defined at $M_X = 1.6 \times 10^{16}$ GeV). Solid lines: complete one-loop results, dashed lines: (renormalization improved) tree level results.

order to get a much more scale independent potential the one-loop correction ΔV_1 is needed. This is given by

$$\Delta V_1 = \sum_j \frac{n_j}{64\pi^2} M_j^4 \left[\log \frac{M_j^2}{Q^2} - \frac{3}{2} \right], \quad (4)$$

where $M_j^2(\phi, t)$ are the tree-level (field-dependent) mass eigenstates and n_j are spin factors. In this way, the minimization of $V = V_o + \Delta V_1$ gives one-loop VEVs $v_1(Q), v_2(Q)$ much more stable against variations of the Q scale. In general, there is a region of Q where $v_1(Q), v_2(Q)$ are remarkably Q -stable and a particular scale, \hat{Q} , always belonging to that region, at which $v_1(\hat{Q}), v_2(\hat{Q})$ essentially coincide with $v_1^o(\hat{Q}), v_2^o(\hat{Q})$. This is illustrated in Fig. 1.

Here we have evaluated v_1, v_2 at \hat{Q} and then obtained $v_1(Q), v_2(Q)$ at any scale by using the RG running of the H_1, H_2 wave functions. Furthermore, we have included in eq. (4) *all* the supersymmetric spectrum since some approximations can lead to wrong results⁹.

Conditions (i)–(iii) allow to eliminate $\alpha_X, M_X, h_t, h_b, h_\tau$, while (iv) allows to eliminate one of the remaining parameters, which we choose to be μ . This finally leaves three independent parameters, namely m, M and A . Actually, for many choices of these parameters there is no value of μ capable of producing

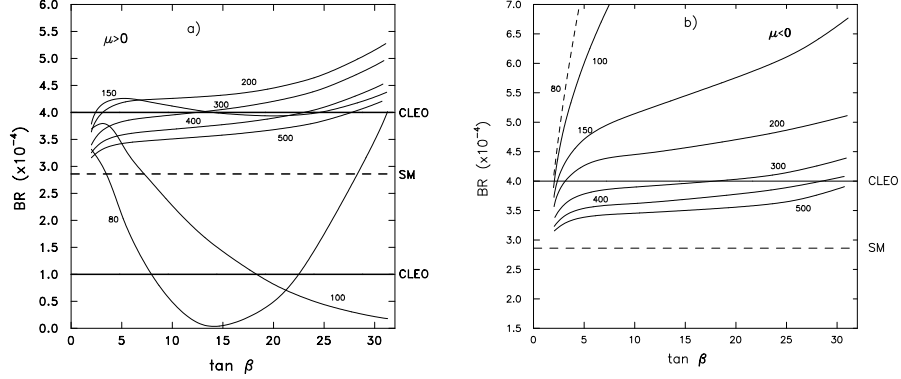


Figure 2: Plot of the branching ratio $BR(b \rightarrow s, \gamma)$ vs $\tan \beta$ for the SUGRA I scenario (minimal SUGRA) with $m = M$ for different values of m (80, 100, 150, 200, 300, 400, 500 GeV) : a) branch $\mu > 0$; b) branch $\mu < 0$. The dashed curves indicate that the model becomes incompatible with the experimental lower bounds on supersymmetric particles. The Standard Model prediction (SM) and the CLEO bounds are also shown in the figure.

the correct electroweak breaking, a feature which restricts the parameter space substantially. On the other hand, there can be two branches of solutions depending on the sign of μ .

2 Results

SUGRA I

As discussed above, our parameter space is now restricted to m, M, A . In order to present the results in a comprehensible way, let us make the assumption $m = M$, and trade the high energy parameter A by the low energy one $\tan \beta = v_2/v_1$. The corresponding plots of the branching ratio $BR(b \rightarrow s, \gamma)$ versus the remaining parameter, $\tan \beta$ for different values of m are given in Fig. 2a (branch $\mu > 0$) and Fig. 2b (branch $\mu < 0$). We can see that for $m \geq 200$ GeV and both $\mu > 0, \mu < 0$, for each value of m there is a maximum acceptable value for $\tan \beta$. In general for $m \geq 200$ GeV the restrictions are stronger for positive values of μ and small values of m . For $m \leq 200$ GeV, however, the situation is different: whereas for $\mu < 0$ the restrictions become very strong (only a narrow range of $\tan \beta$ is allowed), for $\mu > 0$ there appear large windows of allowed values of $\tan \beta$. Here the CLEO lower bound plays a relevant role.

It is clear that the trend to the SM result is so slow that an improvement of the CLEO bounds (particularly the upper one) on BR of order 10^{-4} would push the lower limits on m to the TeV region (except for the above-mentioned

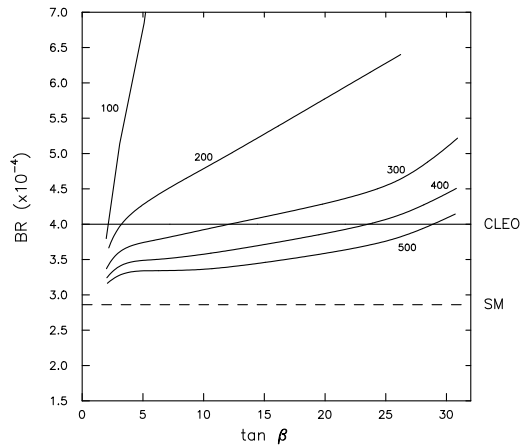


Figure 3: The same as in Fig. 2, but for the SUGRA II scenario.

windows in the $\mu > 0$ case).

SUGRA II

We note here that the sign of μ is always negative since (adopting the convention $v_1, v_2 > 0$) for $\mu > 0$ and $B = 2m$ the necessary electroweak breaking cannot be achieved. The SUGRA II results present a similar pattern to those of SUGRA I with $\mu < 0$. Again, we find from Fig. 3 that for a given value of m there is a maximum acceptable value for $\tan \beta$. The bound becomes less stringent as m increases; therefore, once again, an improvement of the CLEO bounds (especially the upper one) of order 10^{-4} would amount to a dramatical improvement of the MSSM constraints.

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